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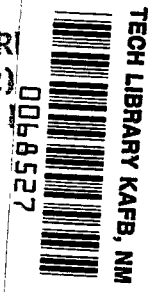

STRESS CONCENTRATION IN A CYLINDRICAL SHELL WITH A ROUND APERTURE IN ITS SIDE

by Yu. A. Shevlyakov

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Translation of "Kontsentratsiya napruzhen' v tsilindrichniy obolontsi z krugovim virizom na bokoviy poverkhni"

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Yu. A. Shevlyakov

ABSTRACT

This study deals with the problem of tension in a cylindrical shell with a round opening in its side, reinforced by a rigid ring around the edge.

The system of partial solutions of equilibrium equations for a cylindrical shell, as proposed by A. I. Lur'ye (Ref. 1), is used in this investigation.

Given a uniform internal pressure, the tension along the welded seam between the shell and the ring can be defined by formulas (3.2).

1. Let us consider the stress in the cylindrical shell with a round hole in its side, reinforced by a rigid ring. The load consists of a uniform internal pressure p_0. We will designate the surface of the shell as polar coordinates ρ, λ. Inside the aperture, $\rho = 0$.

With $\rho = \rho_0$ (ρ_0 is the aperture radius),

$$u_\rho = u_\lambda = \frac{\partial w}{\partial \rho} = 0; \quad w = c. \tag{1.1}$$

The forces and momenta will be (Ref. 1):

$$S_\rho = \frac{1}{2} h (p + q) + \frac{1}{2} h (p - q) \cos 2\lambda -$$

$$- D \frac{1}{h} \sqrt{12 (1 - \nu^2)} \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \lambda^2} \right] \operatorname{Im} \sigma;$$

$$S_{\rho\lambda} = -\frac{1}{2} h (p - q) \sin 2\lambda + \frac{D}{h} \sqrt{12 (1 - \nu^2)} \left[\frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \frac{\partial}{\partial \lambda} \right) \right] \operatorname{Im} \sigma;$$

$$S_\lambda=\frac{1}{2}h(p+q)-\frac{1}{2}h(p-q)\cos 2\lambda-\frac{D}{h}\sqrt{12(1-\nu^2)}\frac{\partial^2}{\partial\rho^2}\operatorname{Im}\sigma; \tag{1.2}$$

$$M_\rho=-D\left(\frac{\partial^2}{\partial\rho^2}+\nu\frac{1}{\rho}\frac{\partial}{\partial\rho}+\nu\frac{1}{\rho^2}\frac{\partial^2}{\partial\lambda^2}\right)\operatorname{Re}\sigma;$$

$$Q_\rho^*=-D\left[\frac{\partial}{\partial\rho}\nabla^2+(1-\nu)\frac{1}{\rho}\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial^2}{\partial\lambda^2}\right]\operatorname{Re}\sigma;$$

$$\nabla^2=\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\lambda^2};$$

$$p=\frac{p_0R}{2h};\quad q=\frac{p_0R}{h},-$$

Where Im σ and $w = \operatorname{Re}\sigma$ will appear as $\left(\beta=\sqrt[4]{3(1-\nu^2)}\,\frac{1}{2\sqrt{Rh}}\right)$, correct to the square of the small parameter β:

$$\operatorname{Im}\sigma=\psi_0+\beta^2\psi_1=\frac{2A_0}{\pi}\left(\ln\frac{\rho}{\rho_0}+\gamma'\right)-\frac{C_0}{\pi}(1+\cos 2\lambda)-$$

$$-\frac{2F_1}{\pi}\frac{\cos 2\lambda}{\rho^2}+\beta^2\left[-\frac{1}{4}A_0\rho^2(2+\cos 2\lambda)+\frac{C_0}{4}\rho^2(1+\cos 2\lambda)+\right. \tag{1.3}$$

$$\left.+\frac{2A_1}{\pi}\left(\ln\frac{\rho}{\rho_0}+\gamma'\right)+\frac{1}{2}B_1-\frac{2F_2}{\pi}\frac{\cos 2\lambda}{\rho^2}-\frac{C_1}{\pi}(1+\cos 2\lambda)\right];$$

$$w=\operatorname{Re}\sigma=\frac{1}{2}A_0+\frac{\beta^2}{\pi}\left\{\frac{\pi A_1}{2}-2B_1\gamma'+D_1-\frac{F_1}{\pi}+\right.$$

$$+A_0\left(2\rho^2\ln\frac{\rho}{\rho_0}+2\rho_0\gamma'-\rho^2\right)-2B_1\ln\frac{\rho}{\rho_0}+$$

$$+C_0\rho^2\left(\frac{1}{4}-\ln\frac{\rho}{\rho_0}-\gamma'\right)+\cos 2\lambda\left[A_0\rho^2\left(\ln\frac{\rho}{\rho_0}+\gamma'\right)+\right.$$

$$+C_0\rho^2\left(\frac{1}{6}-\ln\frac{\rho}{\rho_0}-\gamma'\right)+D_1-\frac{2E_2}{\rho^2}-\left.\frac{4H_2}{\rho^2}\right]-$$

$$\left.-\cos 4\lambda\left(\frac{C_0}{12}\rho^2+\frac{F_1}{2}+\frac{4H_2}{\rho^2}+\frac{24M_3}{\rho^4}\right)\right\}.$$

(In $\gamma = 0.5772$; A_0, C_0, . . . are unknown constants). With $\beta = 0$, the normal curvature w can be considered equal to zero.

2. We will represent the changes u_ρ and u_λ by the solutions (1.3). The relative deformation of the cylindrical surface will be:

$$\varepsilon_\rho=\frac{\partial u_\rho}{\partial \rho}-4hk_1\frac{1}{\sqrt{3(1-\nu^2)}}\beta^2 w;$$

$$\varepsilon_{\rho\lambda}=\frac{\partial u_\rho}{\rho\partial\lambda}+\frac{\partial u_\lambda}{\partial\rho}-\frac{u_\lambda}{\rho}+8hk_2\frac{1}{\sqrt{3(1-\nu^2)}}\beta^2 w; \tag{2.1}$$

$$\varepsilon_\lambda=\frac{\partial u_\lambda}{\rho\partial\lambda}+\frac{u_\rho}{\rho}-4hk_3\frac{1}{\sqrt{3(1-\nu^2)}}\beta^2 w,$$

where k_1, k_2, k_3 are the general functions of angle λ;

$$0\leqslant|k_1|,\ |k_2|,\ |k_3|\geqslant 1.$$

On the other hand, according to Hooke's law:

$$\varepsilon_\rho=\frac{1}{Eh}(S_\rho-\nu S_\lambda);$$

$$\varepsilon_\lambda=\frac{1}{Eh}(S_\lambda-\nu S_\rho); \tag{2.2}$$

$$\varepsilon_{\rho\lambda}=\frac{2(1-\nu^2)}{Eh}S_{\rho\lambda}.$$

On the basis of (1.2), (1.3), (2.1) and (2.2), we will obtain expressions u_ρ, u_λ, correct to the square of β, in the form of:

$$u_\rho=f_1(\rho,\lambda)+\beta^2 f_2(\rho,\lambda);$$
$$u_\lambda=\varphi_1(\rho,\lambda)+\beta^2\varphi_2(\rho,\lambda). \tag{2.3}$$

3. We will substitute u_ρ, u_λ, w, $\frac{\partial w}{\partial\rho}$ in (1.1). Separately equating the coefficients in β^0 and β^2 to 0, we obtain:

$$A_0=-\frac{\pi\rho_0^2(p+q)(1-\nu)}{4Eh(1+\nu)}\sqrt{12(1-\nu^2)};$$

$$F_1=-\frac{\pi\rho_0^4(p-q)(1+\nu)}{8Eh(3-\nu)}\sqrt{12(1-\nu^2)};$$

$$C_0=\frac{\pi\rho_0^2(p-q)(1+\nu)}{2Eh(3-\nu)}\sqrt{12(1-\nu^2)}; \tag{3.1}$$

$$A_1=-\frac{\pi^2\rho_0^4}{8Eh}\sqrt{12(1-\nu^2)}\left[\frac{1-\nu}{1+\nu}(p+q)-\frac{1+\nu}{3-\nu}(p-q)\right]\frac{1-\nu}{1+\nu};$$

$$F_2=-\frac{\pi\rho_0^4}{8}\frac{1+\nu}{3-\nu}(A_0-C_0);$$

$$C_1=-\frac{\pi^2\rho_0^4}{4Eh}\frac{1+\nu}{3-\nu}\sqrt{12(1-\nu^2)}\left[\frac{1+\nu}{3-\nu}(p-q)-\frac{1-\nu}{2(1+\nu)}(p+q)\right].$$

The contact stress on the boundary between the ring and shell will be:

$$\frac{S_\rho}{h}=q\left[1.16-0.37\cos 2\lambda-0.42\frac{\pi\rho_0^2}{Rh}(0.42-0.24\cos 2\lambda)\right];$$

$$\frac{S_{\rho\lambda}}{h}=-q\left(0.37+0.28\frac{\rho_0^2}{Rh}\right)\sin 2\lambda. \qquad (3.2)$$

The first terms in (3.2) conform to the two-dimensional theory of elasticity in the analogous problem (Ref. 2):

$$q=\frac{p_0R}{h},$$

where R denotes the curvature radius and h the shell thickness. In all calculations we used $\nu = 0.3$.

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December 14, 1954

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